Chapter 14

Logical Properties

Logical Properties

Concepts: modeling properties that refer to states

Models: **fluent** – characterization of abstract state based on action sets
fluent linear temporal logic FLTL

Practice: **assert** – Java proposition on the state of variables

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Background

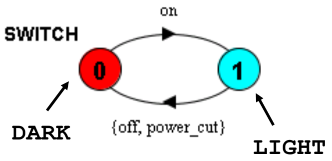
- Temporal Logic due to Pneuli (1977) is a popular means to describe process properties in logic.
- Use propositions on selected variable states at particular points in program executions.
- Realized as the **assert** construct in Java.

States in an LTS model based on actions or events? HOW?

- Introduce **fluents** to describe abstract “states”.
- Express both safety and liveness properties as fluent propositions.

Pnueli, A. (1977). The Temporal Logic of Programs. Proc. of the 18th IEEE Symposium on the Foundations of Computer Science, Oct/Nov 1977, pp. 46-57.
Robert A. Kowalski, Marek J. Sergot (1986). A Logic-based Calculus of Events. New Generation Comput. 4(1): 67-95

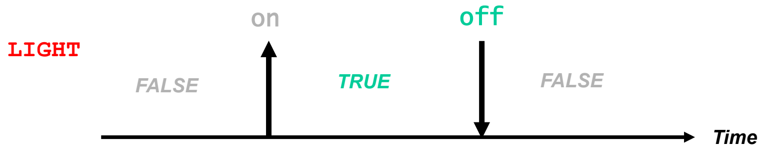
Fluents



```
const False = 0
const True = 1

SWITCH = (on->{off, power_cut}->SWITCH).

fluent LIGHT = <{on},{off, power_cut}>
                   initially False
```

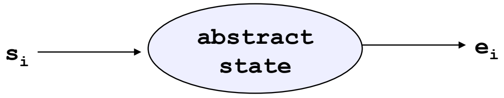


```
fluent DARK = <{off, power_cut},{on}>
                   initially True
```

Fluents

fluent FL = <{s_1,…,s_n},{e_1,…,e_n}> initially *B* defines a fluent FL that is initially true if the expression *B* is true and initially false if the expression *B* is false. FL becomes true when any of the initiating (or starting) actions {s_1,…,s_n} occur and false when any of the terminating (or ending) actions {e_1,…,e_n} occur. If the term **initially** *B* is omitted then FL is initially false. The same action may not be used as both an initiating and terminating action.

A fluent **<{s_1,…,s_n},{e_1,…,e_n}>** thus describes an abstract state that is entered by executing any of the actions in {s_1,…,s_n}, and exited by executing any of the actions in {e_1,…,e_n}.



Fluent Linear Temporal Logic (FLTL) Expressions

- FLTL expression can be constructed using Boolean operators and quantifiers:
  &&, ||, !, ->, <->, forall, exists
- E.g., If the light is on, power is also on:

```
fluent LIGHT = <on, off>
fluent POWER = <power_on, power_off >
LIGHT -> POWER
```

- All lights are on:

```
fluent LIGHT[i:1..2] = <on[i], off[i]>

forall[i:1..2] LIGHT[i]
```

- At least one light is on:

```
fluent LIGHT[i:1..2] = <on[i], off[i]>
exists[i:1..2] LIGHT[i]
```

Fluent Linear Temporal Logic (FLTL) Expressions

- There are five temporal operators in FLTL
  - Always []
  - Eventually <>
  - Until U
  - Weak until W
  - Next time X
- Amongst the five operators, always [] and eventually <> are the two most commonly used ones.
- Until, Weak until and Next time allows complex relation between abstract states.

Temporal propositions

```
const False = 0
const True = 1

SWITCH = (power_on -> OFF),
OFF    = (on -> ON | power_off -> SWITCH),
ON     = (off-> OFF | power_off -> SWITCH).

fluent LIGHT = <on, off>
fluent POWER = <power_on, power_off>

assert OK = [](LIGHT -> POWER)
```

always

implies

Safety Properties: Mutual Exclusion

```
const N = 2
range Int = 0..N
SEMAPHORE(I=0) = SEMA[I],
SEMA[v:Int]    = (up->SEMA[v+1]
                 |when(v>0) down->SEMA[v-1]
                 ).

LOOP = (mutex.down->enter->exit->mutex.up->LOOP).

||SEMADEMO = (p[1..N]:LOOP
             || {p[1..N]}::mutex:SEMAPHORE(2)).

fluent CRITICAL[i:1..N] = <p[i].enter, p[i].exit>
```

◆ Two **processes** are not in their critical sections simultaneously?

Safety Properties: Mutual Exclusion

◆ The linear temporal logic formula []F – always F – is true if and only if the formula F is true at the current instant and at all future instants.

◆ No two processes can be at critical sections simultaneously:

```
assert MUTEX = []!(CRITICAL[1] && CRITICAL[2])
```

◆ LTSA compiles the assert statement into a safety property process with an ERROR state.

Safety Properties: Mutual Exclusion

```
Trace to property violation in MUTEX:
      p.1.mutex.down
      p.1.enter         CRITICAL.1
      p.2.mutex.down    CRITICAL.1
      p.2.enter         CRITICAL.1 && CRITICAL.2
```

◆ General expression of the mutual exclusion property for N processes:

```
 assert MUTEX_N(N=2) = []!(exists [i:1..N-1]
                       (CRITICAL[i] && CRITICAL[i+1..N] ))
```

Safety Properties: Oneway in Single-Lane Bridge

```
const N = 2 // number of each type of car
range ID= 1..N // car identities

fluent RED[i:ID]  = <red[i].enter, red[i].exit>
fluent BLUE[i:ID] = <blue[i].enter, blue[i].exit>

assert ONEWAY  = []!(exists[i:ID] RED[i]
                  && exists[j:ID] BLUE[j])
```

◆ Abbreviating `exists[i:R] FL[i]` as `FL[R]`

```
assert ONEWAY  = []!(RED[ID] && BLUE[ID])
```

Single Lane Bridge - safety property `ONEWAY`

The fluent proposition is more concise as compared with the property process ONEWAY. This is usually the case where a safety property can be expressed as a relationship between abstract states of a system.

```
property ONEWAY =(red[ID].enter   -> RED[1]
                 |blue.[ID].enter -> BLUE[1]
                 ),
RED[i:ID] = (red[ID].enter -> RED[i+1]
            |when(i==1)red[ID].exit  -> ONEWAY
            |when(i>1) red[ID].exit  -> RED[i-1]
            ),         //i is a count of red cars on the bridge
BLUE[i:ID]= (blue[ID].enter-> BLUE[i+1]
            |when(i==1)blue[ID].exit -> ONEWAY
            |when( i>1)blue[ID].exit -> BLUE[i-1]
            ).        //i is a count of blue cars on the bridge
```

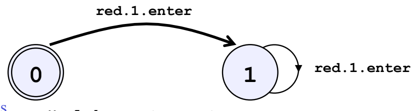
Liveness Properties

The linear temporal logic formula **<>*F*** – eventually *F* – is true if and only if the formula F is true at the current instant or at some future instant.

- First red car must eventually enter the bridge:

```
assert FIRSTRED  = <>red[1].enter
```

- To check the liveness property, LTSA transforms the negation of the assert statement in terms of a **Büchi** automaton.
- A **Büchi** automaton recognizes an infinite trace if that trace passes through an acceptance state infinitely often.



Büchi Automaton

Liveness Properties: Progress Properties

- Compose the Büchi automaton and the original system.
- Search for acceptance state in strong connected components.
- Failure of the search implies no trace can satisfy the Buchi automaton.
- It validates that the assert property holds.
- Red and blue cars enter the bridge infinitely often.

```
assert REDCROSS  = forall [i:ID] []<>red[i].enter
assert BLUECROSS = forall [i:ID] []<>blue[i].enter
assert CROSS     = (REDCROSS && BLUECROSS)
```

Liveness Properties: Response Properties

- If a red car enters the bridge, it should eventually exit.
- It does not stop in the middle or fall over the side!

```
assert REDEXIT = forall [i:ID]
      [](red[i].enter -> <>red[i].exit)
```

- Such kind of properties is sometimes termed “response” properties, which follows the form:

```
[](request-> <>reply)
```

- This form of liveness property cannot be specified using the progress properties discussed earlier.

Fluent Linear Temporal Logic (FLTL)

- There are five operators in FLTL
  - Always []
  - Eventually <>
  - Until U
  - Weak until W
  - Next time X
- Amongst the five operators, always [] and eventually <> are the two most commonly used ones.
- Until, Weak until and Next time allows complex relation between abstract states.

Summary

- A **fluent** is defined by a set of initiating actions and a set of terminating actions.
- At a particular instant, a fluent is true if and only if it was initially true or an initiating action has previously occurred and, in both cases, no terminating action has yet occurred.
- In general, we don't differentiate safety and liveness properties in fluent linear temporal logic FLTL.
- We verify an LTS model against a given set of fluent propositions.
- LTSA evaluates the set of fluents that hold each time an action has taken place in the model.

Course Outline

2. **Processes and Threads**
3. **Concurrent Execution**
4. **Shared Objects & Interference**
5. **Monitors & Condition Synchronization**
6. **Deadlock**
7. **Safety and Liveness Properties**
8. **Model-based Design (Case Study)**

The main basic

Concepts

Models

Practice

Advanced topics …

9. Dynamic systems
10. Passing
11. Concurrent Software Architectures
12. Timed Systems
13. Program Verification
14. **Logical Properties**